EP-94

NET PRESENT VALUE UNDER FINANCING CONDITIONS

Rodrigo Varela V.
ICESI- UNIVALLE

1. INTRODUCTION

For most of the investment projects that an investor will consider, there is practically always an alternative to marshal the financial resources: the use of a loan or any other external financing scheme.

Undoubtedly the investor should study the economic and financial conditions of the loan to define if it is to his advantages to use it or not. Usually this process requires to evaluate the project under own resources (0% financing) and to evaluate it under loan conditions (a percentage of the resources came from a loan), and from these two results a decision is made about the advantages or disadvantages the loan has on the economic indicator of the project. This procedure is long and very frequently many conceptual mistakes are done in the development and in the interpretation of results (Reference 1 shows in detail these mistakes).

In a previous paper (Reference 2), a deterministic simulation was done for a particular example and a mathematical model was developed showing general forms for the economic criteria as the following ones:

$$IRR_f - IRR_c = f\,(H,K,i^*) \qquad (1)$$

$$NPV_f - NPV_c = g\,(H,K,i^*) \qquad (2)$$

Where :

IRR_C, IRR_f = Internal rate of return of the project under 100% own resources (cash investment) and under financing conditions (Leveraged investment)

NPV_C, NPV_f = Net present value at the minimum attractive rate of return (i*), under 100% own resources (cash investment) and under financing condition (leveraged investment)

i* = Minimum after tax attractive rate of return

After tax capital cost

Fraction of the initial investment obtained as loan (Leveraged fraction)

In this article the goal is to develop an analytical model that allows the investor to predict easily the real effect that on the NPV will have a particular financial scheme (K, H, M), allowing him to make in an easier form sensitivity analysis about these variable and to take better decisions. Additionally, the development of the model will allow to study analytically the effect of the different variables involved in the problem.

2. METHODOLOGY

It is well known that to do a real after tax investment evaluation it is necessary to have the net own Investment (INP) and the net cash flow (FCN) of the investor through the analysis period (N) to get some economic criteria (NPV, IRR, etc.), using the minimum after tax attractive rate of return.

For the analysis with loan it is necessary to do the analysis under cash investment and under the leveraged investment. Thus the general methodology was:

a) Development of an analytical model for the totally net cash flow (Net cash flow minus net own investment)

b) Development of analytical models for the Net Present Value (NPV) for the cash and the leveraged investment

c) Development of an analytical model for the leveraged NPV as a function of the cash NPV and other variables of the project and of the financing scheme

d) Interpretation of results

3. GENERAL MODEL FOR THE TOTALLY NET CASH FLOWS

The totally net cash flows are defined as the difference between the net cash flows and the net own investment increased or decreased by other cash flows (market value of equipment, working capital recovery, etc.) . Figure 1 present a very concise process to get totally net cash flows.

It is understood that in the calculation of earnings before taxes and interest all the other costs and the tax deductions (depreciation, amortization, depletion, etc.), had been deducted.Also that in the item total investment all the fix investments, the working capital and the pre-operating expenses have been included.

	EARNINGS BEFORE INTERESTS AND TAXES
-	INTERESTS
	NET OPERATING INCOME
-	TAXES
	NET PROFIT AFTER TAXES
+	TAX DEDUCTIONS
-	CAPITAL PAYMENTS
1	**NET CASH FLOW**
	INVESTMENTS
-	LOAN
2	**NET INVESTMENT**
3	**OTHER CASH FLOWS**
1-2+3	**TOTAL NET CASH FLOW**

Figure 1

Thus the big difference between cash evaluation (100% of the total investment are done with own resources) and the leveraged evaluation (H % of the total investments is done with external resources : loans) are in the following three aspects: loans, interestS, capital payments.

If the following definitions are made:

$UAII_j$ = Earnings before taxes and interest in year "j"

D_j = Tax deductions in the year "j"

I_o = Total initial investment in "o"

I_j = Total investment in "j"
H = Fraction of I_o, which is obtained by a loan in "o"

VR_j = Other Cash flows in year "j"

L = Yearly cost of capital before taxes

M = Number of years to pay the loan

r = Tax rate

It is very easy to get the following results:

$$\text{Total loan} = P = H \cdot I_o \tag{3}$$

$$\text{Yearly Principal Payment} = \frac{P}{M} = \frac{H \cdot I_o}{M} \tag{4}$$

$$\text{Balance of loan at the end of year "j"} = S_j = H \cdot I_o \cdot \left(1 - \frac{j}{M}\right) \tag{5}$$

$$\text{Interests in year "j"} = S_j \cdot L = H \cdot I_o \cdot L \cdot \left(1 - \frac{(j-1)}{M}\right) \tag{6}$$

With these calculations and Figure 1 it is possible to obtain the following expressions for the yearly totally net cash flow :

3.1. For the cash situation:

i) for j = 0

$$FCTN_{c,o} = -I_o \tag{7}$$

ii) for j = 1, N

$$FCTN_{c,j} = UAII_j \cdot (1-r) + D_j - I_j + VR_j \tag{8}$$

3.2. For the leveraged situation:

i) for j = 0

$$FCTN_{f,j} = -I_o + H \cdot I_o = -I_o \cdot (1-H) \tag{9}$$

ii) for j = 1, M

$$FCTN_{f,j} = UAII_j \cdot (1-r) - H \cdot I_o \cdot L \cdot \left(1 - \frac{(j-1)}{M}\right)(1-r) + D_j - I_j - \left(\frac{H \cdot I_o}{M}\right) + VR_j \tag{10}$$

iii) for j = M +1,N

$$FCTN_{f,j} = UAII_j \cdot (1-r) + D_j - I_j + VR_j \tag{11}$$

And from them, we can deduct:

a) for j = 0

$$FCTN_{f,o} = FCTN_{c,o} + H \cdot I_o \tag{12}$$

b) for j = 1, M

$$FCTN_{f,j} = FCTN_{c,j} - H \cdot I_o \cdot L \cdot \left(1 - \frac{(j-1)}{M}\right)(1-r) - \frac{H \cdot I_o}{M} \tag{13}$$

c) for j = M + 1, N

$$FCTN_{f,j} = FCTN_{c,j} \tag{14}$$

Equation 13 could be rearranged as follows:

$$FCTN_{f,j} = FCTN_{c,j} - \frac{H \cdot I_o \cdot L}{M}\left[(M+1)(1-r) + \frac{1}{L}\right] + \frac{H \cdot I_o \cdot L}{M}(1-r) \cdot j \tag{15}$$

The second member of the right hand side of equation 15 is a constant for the years 1 through M, whereas the third member is a function of the year "j"

There is a very special situation in equations 12, 14 and 15 and it is that the totally net cash flow for the leveraged case is a function of the totally net cash flow for the cash case and two other types of variable : the financing ones (H, L, M) and the project ones (Io, r, N)

4. GENERAL NPV MODEL

It is well known that the Net Present Value can be calculated by the following equation

$$NPV_{i*} = \sum_{j=0}^{N}\left(\frac{FCTN_j}{(1+i*)^j}\right) \qquad (16)$$

In particular, for the cash situation, the result is:

$$NPV_{c,i*} = -I_o + \sum_{j=1}^{N}\frac{FCTN_{c,j}}{(1+i*)^j} \qquad (17)$$

And for the leveraged situation the result is:

$$NPV_{f,i*} = -I_o + H\cdot I_o + \sum_{j=1}^{M}\frac{FCTN_{f,j}}{(1+i*)^j} + \sum_{j=M+1}^{N}\frac{FCTN_{f,j}}{(1+i*)^j} \qquad (18)$$

When equations 14 and 15 are used in equation 18, the following result is obtained:

$$NPV_{f,i*} = -I_o + H\cdot I_o + \sum_{j=1}^{M}\frac{FCTN_{c,j}}{(1+i*)^j} + \sum_{j=M+1}^{N}\frac{FCTN_{c,j}}{(1+i*)^j} - \sum_{j=1}^{M}\frac{\frac{H\cdot I_o\cdot L}{M}\left[(M+1)(1-r)+1/L\right]}{(1+i*)^j} + \sum_{j=1}^{M}\frac{H\cdot I_o\cdot L\cdot(1-r)\cdot j}{M\cdot(1+i*)^j} \qquad (19)$$

If we substitute the following definition:

$$\sum_{j=1}^{M}\frac{FCTN_{c,j}}{(1+i^*)^j}+\sum_{j=M+1}^{N}\frac{FCTN_{c,j}}{(1+i^*)^j}=\sum_{j=1}^{N}\frac{FCTN_{c,j}}{(1+i^*)^j} \quad (20)$$

in equation 17 we get:

$$NPV_{c,i^*}=-I_o+\sum_{j=1}^{M}\frac{FCTN_{c,j}}{(1+i^*)^j}+\sum_{j=M+1}^{N}\frac{FCTN_{c,j}}{(1+i^*)^j} \quad (21)$$

Now, given that there is a constant quantity:

$$H\cdot I_o\cdot L\cdot[(M+1)(1-r)+1/L)]$$

at the end of all and each one of the years between O and M and that:

$$(P/A,i^*,M)=\sum_{j=1}^{M}\frac{1}{(1+i^*)^j}$$

it is possible to get the following simplification:

$$\sum_{j=1}^{M}\frac{\frac{H\cdot I_o\cdot L}{M}[(M+1)(1-r)+1/L]}{(1+i^*)^j}=\frac{H\cdot I_o\cdot L}{M}[(M+1)(1-r)+1/L]\,(P/A,i^*,M) \quad (22)$$

And using equation 21 and 22 in equation 19 we get:

$$NPV_{f,i^*} = NPV_{c,i^*} + H \cdot I_o - \frac{H \cdot I_o \cdot L}{M}\left[(M+1)(1-r)+1/L\right] (P/A,i^*,M) + \frac{H \cdot I_o . L(1-r)}{M}\sum_{j=1}^{M}\frac{j}{(1+i^*)^j} \tag{23}$$

which could be rearranged as:

$$NPV_{f,i^*} - NPV_{c,i^*} = \frac{H \cdot I_o \cdot L}{M}\left\{\frac{M}{L} - \left[(M+1)(1-r)+\frac{1}{L}\right](P/A,i^*,M + (1-r)\sum_{j=1}^{M}\frac{j}{(1+i^*)^j}\right\} \tag{24}$$

or as:

$$NPV_{f,i^*} - NPV_{c,i^*} = H \cdot I_o\left\{1 - \frac{(P/A,i^*,M)}{M} - \frac{L}{M}(M+1)(1-r) (P/A,i^*M) + \frac{L}{M}(1-r)\sum_{j=1}^{M}\frac{j}{(1+i^*)^j}\right\} \tag{25}$$

or as:

$$NPV_{f,i^*} - NPV_{c,i^*} = H \cdot I_o \left\{ \left(1 - \frac{(P/A,i^*,M)}{M}\right) - L(1-r)\left(\frac{(M+1)}{M} (P/A,i^*,M) - \frac{1}{M}\sum_{j=1}^{M} \frac{j}{(1+i^*)^j} \right) \right\} \quad (26)$$

In Annex #1, it is shown that:

$$\left(\frac{M+1}{M}\right)(P/A,i^*,M) - \frac{1}{M}\sum_{j=1}^{M}\frac{j}{(1+i^*)^j} = \left(1 - \frac{(P/A,i^*,M)}{M}\right)\frac{1}{i^*} \quad (27)$$

so equation 26 could be written as:

$$NPV_{f,i^*} - NPV_{c,i^*} = H \cdot I_o \cdot \left\{ \left(1 - \frac{(P/A,i^*,M)}{M}\right) - L(1-r) \left(1 - \frac{(P/A,i^*,M)}{M}\right)\frac{1}{i^*} \right\} \quad (28)$$

or:

$$NPV_{f,i^*} - NPV_{c,i^*} = \frac{H \cdot I_o}{i^*}\left(1 - \frac{(P/A,i^*,M)}{M}\right)\left[i^* - L(1-r)\right] \quad (29)$$

Equation 29 shows a very important result; the difference between the Net Present Value under leveraged conditions and the Net Present Value under cash condition is a function of the two types of variables we have identify in equation 12 an 14, but also it is a function of the minimum attractive rate of return (i^*)

If we define :

$$C = \frac{I_o}{i^*}\left(1 - \frac{(P/A,i^*,M)}{M}\right) \tag{30}$$

and we use K as the after tax cost of the loan given by the expression

$$K = L(1-r) \tag{31}$$

Equation 29 could be written as:

$$NPV_{f,i^*} - NPV_{c,i^*} = C \cdot H \cdot (i^* - K) \tag{32}$$

which is the generic model, that we were looking for, and that shows that the difference between the Net Present Value for the leveraged and cash situation is a function of:

a) Intrinsic variables of the project, represented in the model by the constant C, which include the initial total investment (Io) the after tax minimum attractive rate of return (i^*) and the loan payment period.(M).

b) Financing variables in terms of the amount of leverage used (H)

c) The difference between the after tax minimum attractive rate of return (i^*) and the after tax cost of the loan (K)

5. INTERPRETATION OF RESULTS

Given the proportionality of the analytical model developed, it is very straight forward to get some general rules.

a) The Net Present Value of the leveraged situation will be better than the Net Present Value of the cash situation if and only if the after tax minimum attractive rate of return (i^*) is greater than the after tax cost of the loan (K). This difference is the cause of the effects of the leverage. In general, is good to use loans if it is after tax cost is lower that the after tax minimum rate of return.

b) The amount of leverage (H) works as amplifier of the cause; if the cause is positive a higher H will get a higher NPV but if the cause is negative a lower NPV, will be obtained

c) Even though in the definition of the constant C, there is a financing variable M, the most important effect comes from Io and i^*

d) It is obvious that if, $i^* = K$ a very frequent consideration in finance, there is not economic advantage in using the leveraged situation.

e) Also if a good financial deal is made ($K < i^*$) it is possible that a project with a negative NPV in the cash situation (non feasible project) could present a positive NPV at the leveraged situation (feasible project). Here a deep analysis of the risk conditions on both cases should be included before taking a final decision about the project. Vice versa if a bad financial deal is make ($k > i^*$) it is possible that a good project ($NPV_C > 0$) could be affected and became a non feasible project under financial conditions. ($NPV_f < 0$). Here again a more detailed analysis should be conducted specially for finding a good financial agreement.

6. CONCLUSIONS

A analytical model to predict the Net Present Value of a leverage project was developed as a function of the financial variables (H, K, M) and the project variables (Io, i^*, r, NPV_C)

This model will make very easy the analysis and the explanation of the effect of financing in the economic result of the project under different financial conditions.

$$NPV_f = NPV_c + C \cdot H \cdot (i^* - K)$$

7. REFERENCES

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ANNEX # 1

$$\left(\frac{M+1}{M}\right)(P/A,i^*,M)-\frac{1}{M}\sum_{j=1}^{M}\frac{j}{(1+i^*)^j}=$$

$$\left(\frac{M+1}{M}\right)(P/A,i^*,M)-\frac{1}{M}\left(\frac{1+i^*}{i^*}\right)\left[(P/A,i^*,M)-\frac{M}{(1+i^*)^{M+1}}\right]=$$

$$\left(\frac{M+1}{M}\right)(P/A,i^*,M)-\left(\frac{1+i^*}{i^*}\right)\left(\frac{(P/A,i^*,M)}{M}\right)+\frac{1}{(1+i^*)^{M+1}}\left(\frac{1+i^*}{i^*}\right)=$$

$$(P/A,i^*,M)+\left(\frac{P/A,i^*,M}{M}\right)-\left(\frac{1+i^*}{i^*}\right)\frac{(P/A,i^*,M)}{M}+\frac{1}{i^*\cdot(1+i^*)^M}=$$

$$(P/A,i^*,M)+\left(\frac{P/A,i^*,M}{M}\right)\left[\frac{i^*-1-i^*}{i^*}\right]+\frac{1}{i^*\cdot(1+i^*)^M}=$$

$$\frac{(1+i^*)^M-1}{i^*\cdot(1+i^*)^M}-\frac{(P/A,i^*,M)}{M\cdot i^*}+\frac{1}{i^*\cdot(1+i^*)^M}=$$

$$\frac{(1+i^*)^M}{i^*\cdot(1+i^*)^M}-\frac{(P/A,i^*,M)}{M\cdot i^*}=$$

$$\frac{1}{i^*}-\frac{(P/A,i^*,M)}{M\cdot i^*}=\frac{1}{i^*}\left(1-\frac{(P/A,i^*,M)}{M}\right)$$

THUS:

$$\left(\frac{M+1}{M}\right)(P/A,i^*,M)-\frac{1}{M}\sum_{j=1}^{M}\frac{j}{(1+i^*)^j}=\frac{1}{i^*}\left(1-\frac{(P/A,i^*,M)}{M}\right)$$